12.6: Curvilinear motion: normal and tangential components

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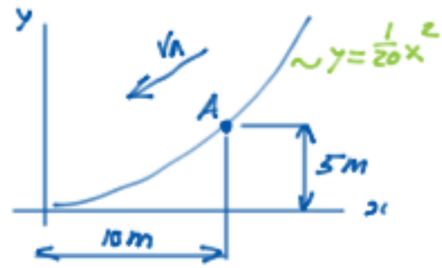
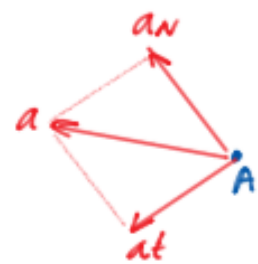
Page 48, example 12-14: A skier travels with a constant speed of 6m/s along the parabolic path y=(1/20)x^2 shown. Determine his velocity and acceleration at the instant he arrives at A. Neglect the size of the skier in the calculation.



Velocity is constant so @ point A $v = 6\,m/s$

acceleration @ A : $a = \sqrt{a_N^2 + a_t^2}$

$$a_N = \frac{v^2}{\rho} = \frac{6^2}{\rho}$$



$$\left.\begin{aligned} \frac{dx}{dy} &= \frac{1}{10}x \\ \frac{d^2y}{dx^2} &= \frac{1}{10} \end{aligned}\right\} \rho = \frac{\left(1+\left(\frac{1}{10}x\right)^2\right)^{1.5}}{\frac{1}{10}}$$

$$\rho(10) = \frac{(1+(1)^2)^{1.5}}{\frac{1}{10}} = 28.28\ m$$

Curvature formula:

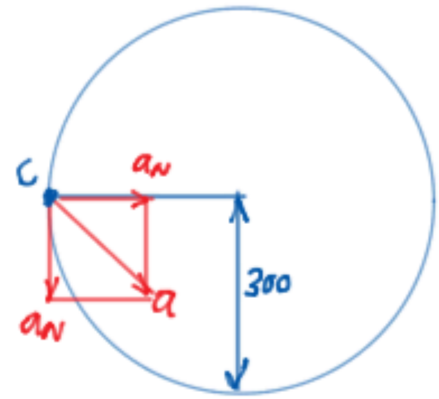
$$\boxed{\rho = \left| \frac{\left(1+\left(\frac{dx}{dy}\right)^2\right)^{3/2}}{d^2y/dx^2} \right|}$$

$$a_N = \frac{6^2}{28.28} = 1.27\ m/s^2$$

$a_t = 0$ (since v=constant)

$$a = \sqrt{(1.27)^2 + 0^2} = 1.27\ m/s^2$$

Page 49, example 12-15: A race car C travels around the horizontal circular track that has a radius of 300ft. If the car increases its speed at a constant rate of 7ft/s^2, starting from rest, determine the time needed for it to reach an acceleration of 8ft/s^2. What is it's speed at this instant?



$$a_t = 7 ft/s^2$$

$$a_N = \frac{v^2}{\rho} = \frac{v^2}{300}$$

$$v = a_t \cdot t$$

$$v = 7 \cdot t$$

$$a_N = \frac{7t^2}{300}$$

$$a_N = 0.163 t^2$$

$$a = \sqrt{a_N^2 + a_t^2}$$

$$8 = \sqrt{(0.163t^2)^2 + 7^2}$$

$$64 = (0.163t^2)^2 + 49$$

$$15 = (0.163 t^2)^2$$

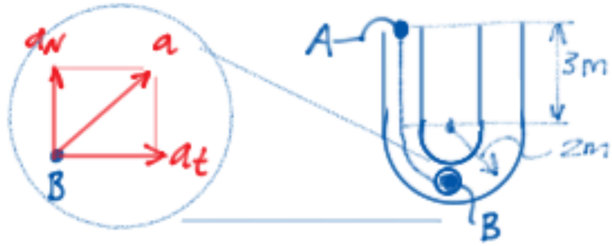
$$3.87 = 0.163 t^2$$

$$t^2 = 23.76$$

$$t = 4.87 s \rightarrow v = a \cdot t$$

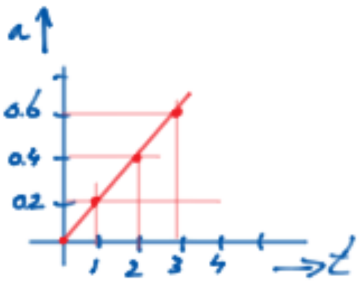
$$= 7 \cdot 4.87 = 34.12 \ ft/s$$

Page 50, example 12-16: A car starts from rest at point A and travels along the horizontal track shown. During the motion, the increase in speed is at=0.2t m/s^2 where t is in seconds. Determine the magnitude of the car's acceleration when it arrives at point B.



$a_t = 0.2t \ m/s^2$

track length is $3m + \frac{2\pi \cdot 2}{4} = 6.14 \ m$



$$\left.\begin{array}{l} a = \frac{dv}{dt} \\ a = 0.2t \end{array}\right\} \begin{array}{l} 0.2t = \frac{dv}{dt} \\ dv = 0.2t \cdot dt \\ v = 0.1t^2 \end{array}$$

$$\Rightarrow \left.\begin{array}{l} v = \frac{ds}{dt} \\ v = 0.1t^2 \end{array}\right\} \begin{array}{l} 0.1t^2 = \frac{ds}{dt} \\ ds = 0.1t^2 \cdot dt \\ s = 0.03t^3 \end{array}$$

@ B, s = 6.14 : $6.14 = 0.03t^3$

$t^3 = 184.2$

$t = 5.69$ sec

@ $t = 5.69$, $v = 0.1t^2$

$= 0.1 \cdot 5.69^2$

$= 3.23 \ m/s$

$$a_N = \frac{v^2}{\rho} = \frac{3.23^2}{2} = 5.24 \ m/s^2$$

$$a_t = 0.2 \cdot t = 0.2 \cdot 5.69 = 1.14 \ m/s^2$$

$$a = \sqrt{a_N^2 + a_t^2}$$

$$= \sqrt{5.24^2 + 1.14^2}$$

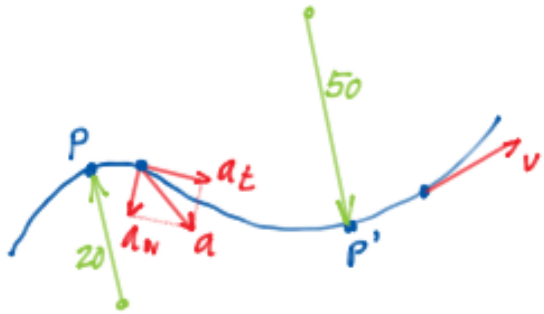
$$= 5.36 \ m/s^2$$

Page 51, problem 12-93: A particle is moving along a curved path at a constant speed of 60ft/s. The radii of curvature of the path at points P and P' are 20 and 50 ft, respectively. If it takes the particle 20 sec to go from P to P', determine the acceleration of the particle at P and P'.

$V = 60 ft/s =$ constant, meaning tangential acceleration (a_t) is 0.

$@P, \ a_N = \frac{V^2}{\rho} = \frac{60^2}{20} = 180 \ ft/s^2$

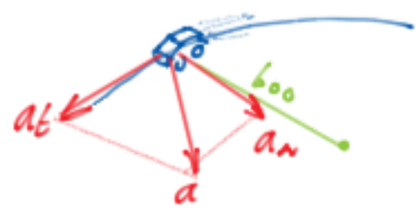
$@P', \ a_N = \frac{V^2}{\rho} = \frac{60^2}{50} = 72 \ ft/s^2$



Page 51, 12-94: A car travels along a horizontal curved road that has a radius of 600m. If the speed is uniformly increased at a rate of 2000km/h^2, determine the magnitude of the acceleration at the instant the speed of the car is 60km/h.

$60 km/h = \frac{60.000 \ m}{3600 \ s} = 16.67 \ m/s$

$2000 \ km/h^2 = \frac{2000.000 \ m}{3600 \times 3600 \ s} = 0.15 \ m/s^2$



$a_t = 0.15 \ m/s^2 =$ constant

$a_N = \frac{V^2}{\rho} = \frac{16.67^2}{600} = 0.46 \ m/s^2$

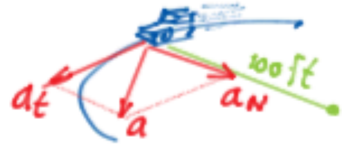
$a = \sqrt{(a_t^2) + (a_N)^2} = \sqrt{(0.15^2) + (0.46^2)} = 0.47 \ m/s^2$

Page 51, problem 12-95: A boat is traveling along a circular path having a radius of 20m. Determine the magnitude of the boat's acceleration if at a given instant the boat's speed is v=5 m/s and the rate of increase in the speed is dv/dt =2 m/s^2.

$$a = \sqrt{(a_t^2) + (a_N^2)}$$
$$a_t = \frac{dv}{dt} = 2\ m/s^2$$
$$a_N = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25\ m/s^2$$

$$a = \sqrt{(2^2) + (1.25^2)}$$
$$a = 2.36\ m/s^2$$

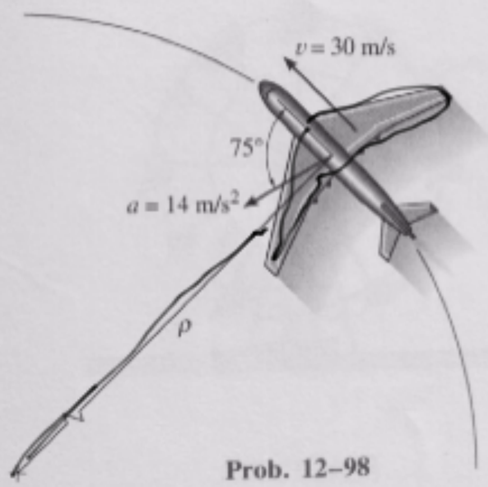
Page 51, problem 12-97: A car moves along a circular track of radius 100ft such that it's speed for a short period of time 0 <= t <= 4s is v=3(t+t^2) ft/s, where t is in seconds. Determine the magnitude of its acceleration when t= 2s. How far has the car traveled in 2s ?



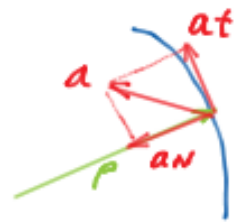
$$V(t) = 3(t + t^2)$$
$$V(2) = 3(2 + 2^2) = 18\ ft/s$$
$$a_N = \frac{v^2}{\rho} = \frac{18^2}{100} = 3.24$$
$$a_t = \frac{dv}{dt} = \frac{d(3t + 3t^2)}{dt} = 3 + 6t \Rightarrow a_{t(2)} = 3 + 6 \cdot 2 = 15$$
$$a = \sqrt{(a_N^2) + (a_t)^2} = \sqrt{(3.24^2) + (15^2)} = 15.3\ ft/s^2$$

$$v = \frac{ds}{dt}$$
$$3(t + t^2) = \frac{ds}{dt}$$
$$ds = 3(t + t^2) \cdot dt$$
$$= 3t \cdot dt + 3t^2 \cdot dt$$
$$s = 1.5t^2 + t^3$$
$$S(2) = 1.5 \cdot 4 + 8 = 14\ ft$$

12–98. At a given instant the airplane has a speed of 30 m/s and an acceleration of 14 m/s² acting in the direction shown. Determine the rate of increase in the plane's speed and the radius of curvature of the path.



Prob. 12–98



$a_N = a \cdot \sin 75^\circ = 14 \cdot 0.966 = 13.52 \ m/s^2$

$a_t = a \cdot \cos 75^\circ = 14 \cdot 0.259 = \boxed{3.62 \ m/s^2}$

$a_N = \frac{v^2}{\rho} \Rightarrow 13.52 = \frac{30^2}{\rho} \qquad \rho = \frac{30^2}{13.52}$

$\rho = \boxed{66.57 \ m}$

Page 51, problem 12-99: A race car has an initial speed of V0=15m/s when s=0. If it increases its speed along the circular track at the rate of at=(0.4s) m/s^2, where s is in meters, determine the normal and tangential components of the car's acceleration when s=100m.

$V_0 = 15 \ m/s$

$a_t = 0.4s \ m/s^2$

$@ s = 100 m \Rightarrow a_t = 0.4 \cdot 100$
$= \boxed{40 \ m/s^2}$

$v = \frac{ds}{dt}, \quad a = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a}$

$v = \frac{ds}{(dv/a)} \rightarrow ds = v \cdot \frac{dv}{a}$

$ds \cdot a = v \cdot dv$

$0.4s \cdot ds = v \cdot dv$

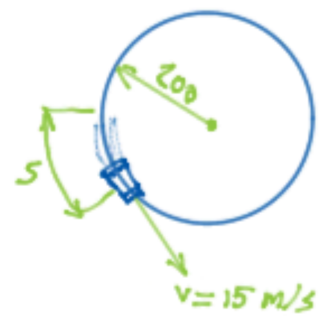
$0.2s^2 = \frac{1}{2}v^2 + C$

$@ s = 0, \ v = 15 \Rightarrow C = -112.5$

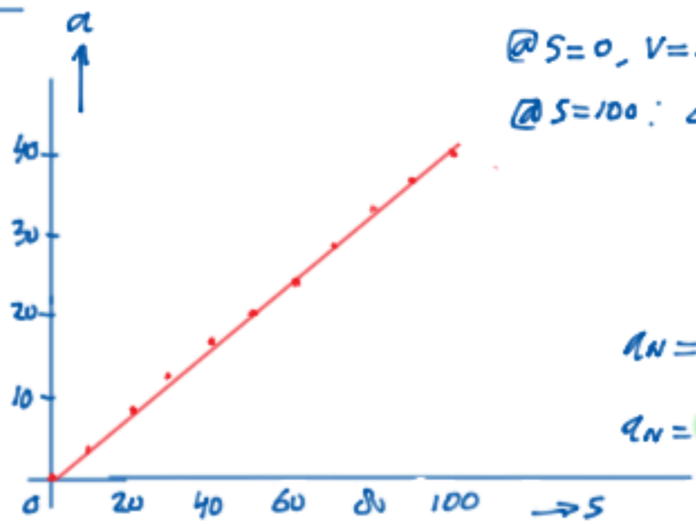
$@ s = 100: \ 0.2 \cdot 100^2 = \frac{1}{2}v^2 - 112.5$

$2000 = \frac{1}{2}v^2 - 112.5$

$v = 65 \ m/s$

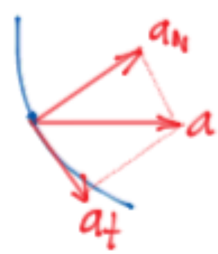


s	a
0	0
10	4
20	8
30	12
40	16
50	20
60	24
70	28
80	32
90	36
100	40

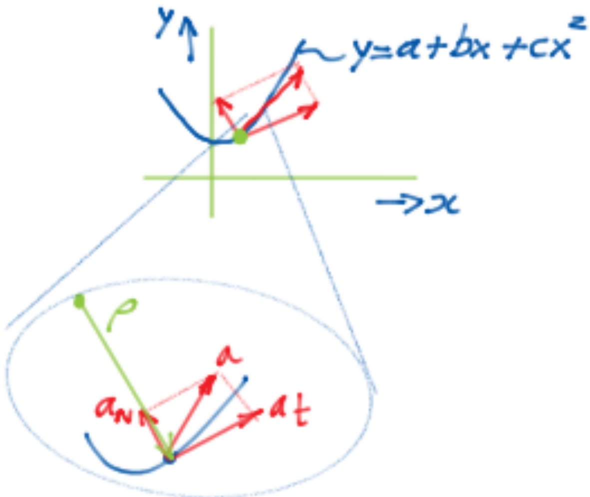


$a_N = \frac{v^2}{\rho} = \frac{65^2}{200}$

$a_N = \boxed{21.13 \ m/s^2}$



Page 51, problem 12-101: A particle travels along the path y=a+bx+cx^2, where a, b, c are constants. If the speed of the particle is constant, v=v0, determine the x and y components of velocity and the normal component of acceleration when x=0.



$a_t = 0$ since speed is constant

$a_N = \frac{v^2}{\rho}$

$y = a + bx + cx^2$

$\dot{y} = b + 2cx \rightarrow @x=0, \; \dot{y} = b$

$\ddot{y} = 2c \rightarrow @x=0, \; \ddot{y} = 2c$

$$\rho = \left| \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} \right|$$

$$= \left| \frac{(1+b^2)^{3/2}}{2c} \right|$$

$$a_N = \frac{v^2}{\frac{(1+b^2)^{3/2}}{2c}} = \frac{2c \cdot v^2}{(1+b^2)^{3/2}}$$

$$a_N = \frac{2 \cdot c \cdot v^2}{(1+b^2)^{3/2}}$$

Page 51, problem 12-103: The motorcyclist travels along the curve at a constant speed of 30ft/s. Determine his acceleration when located at point A. Neglect the size of the motorcycle and rider for the calculation.

$v = 30 ft/s$ (constant)

$y = \frac{500}{x}$ (path curve)

$= \frac{1}{x} \cdot 500$

$= x^{-1} \cdot 500$

$dy/dx \downarrow$

$\dot{y} = -x^{-2} \cdot 500$

at $x = 100 ft$

$\dot{y} = -100^{-2} \cdot 500$

$= 0.05$

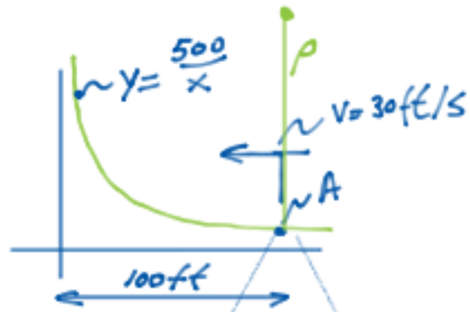
$d^2y/dx^2 \downarrow$

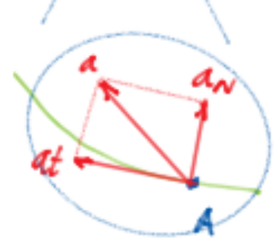
$\ddot{y} = 2x^{-3} \cdot 500$

at $x = 100 ft$

$\ddot{y} = 2 \cdot 100^{-3} \cdot 500$

$= 0.001$





$a_t = 0$ (since v = constant)

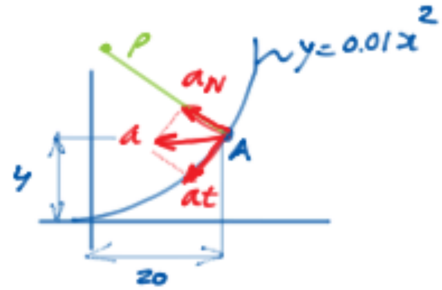
$a_N = \frac{v^2}{\rho}$

$$\rho = \left| \frac{(1 + (dy/dx)^2)^{3/2}}{d^2y/dx^2} \right| = \left| \frac{(1 + (0.05)^2)^{3/2}}{0.001} \right| = 1003.75$$

$$a_N = \frac{30^2}{1003.75} = 0.897 \ m/s^2$$

$$a = \sqrt{a_N^2 + a_t^2} = 0.897 \ m/s^2$$

Page 52, problem 12-105: A bicycle B is traveling down along a curved path which can be approximated by the parabola y=0.01x^2. When it is at A (20,4), the speed of B is measured as v=8m/s and the increase in speed is dv/dt = 4m/s^2. Determine the magnitude of the acceleration of bicycle B at this instant. Neglect the size of the bicycle.



$y = 0.01x^2$

@B the speed of A, $v_A = 8\ m/s$

@B the acceleration, $a_t = \frac{dv}{dt} = 4\ m/s^2$

@B, $a_N = \frac{v^2}{\rho} = \frac{8^2}{\rho}$

$$\rho = \left| \frac{(1+(dy/dx)^2)^{3/2}}{(d^2y/dx^2)} \right|$$

$\frac{dy}{dx} = 0.02x \Rightarrow$ when $x = 20$, $\frac{dy}{dx} = 0.02 \cdot 20 = 0.4$

$\frac{d^2y}{dx^2} = 0.02$

$$\rho = \left| \frac{(1+(0.4)^2)^{3/2}}{0.02} \right|$$

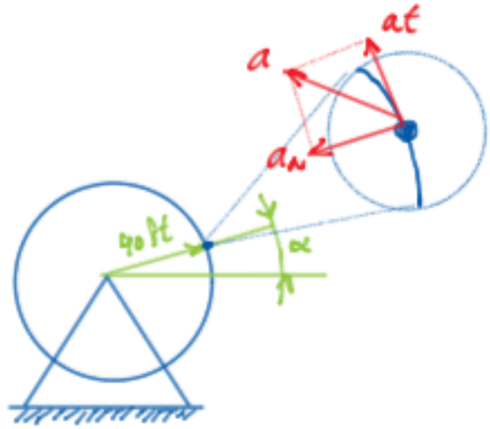
$\rho = 62.47$

$a_N = \frac{8^2}{62.47} = 1.02\ m/s^2$

$a = \sqrt{a_N^2 + a_t^2}$

$= \sqrt{1.02^2 + 4^2} = 4.13\ m/s^2$

Page 52, problem 12-107: The ferris wheel turns such that the speed of the passengers is increased by v'=(4t)ft/s^2, where t is in seconds. If the wheel starts from rest when alfa=0deg, determine the magnitude of the velocity and acceleration of the passengers when the wheel turns alfa=30deg.



$\dot{v} = a_t = 4t \ ft/s^2$

@ $t=0$, $\alpha = 0°$

@ $\alpha = 30°$, distance traveled s is: $s = \frac{2\pi \cdot R}{360} \times \alpha$

$s = \frac{2 \cdot \pi \cdot 40}{360} \times 30$

$= 20.94 \ ft$

$\left.\begin{array}{l} a = 4t \\ a = \frac{dv}{dt} \end{array}\right\} \frac{dv}{dt} = 4t$

$dv = 4t \cdot dt$

$\left.\begin{array}{l} v = 2t^2 \\ v = \frac{ds}{dt} \end{array}\right\} \frac{ds}{dt} = 2t^2$

$ds = 2t^2 \cdot dt$

$s = \frac{2}{3}t^3 \longrightarrow 20.94 = \frac{2}{3}t^3$

$t^3 = 31.41$

$t = 3.16 \longrightarrow v = 2t^2$

$= 2 \cdot 3.16^2$

$= 19.9 \ m/s$

$a_t = 4 \cdot t = 4 \cdot 3.16 = 12.62$

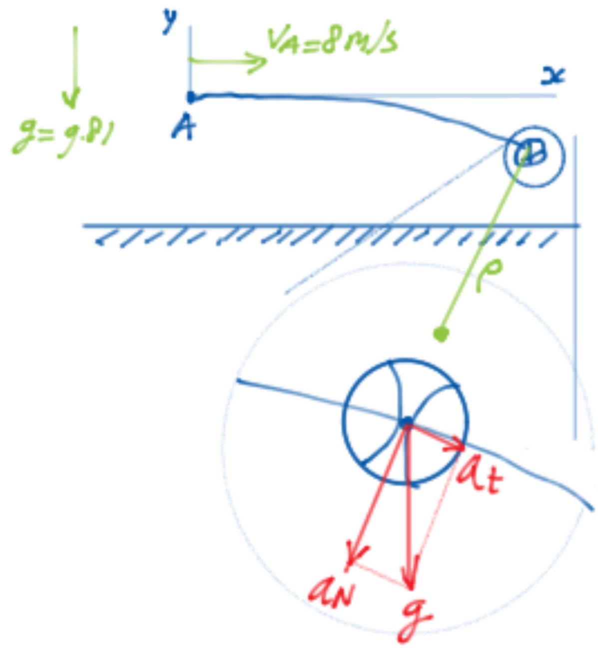
$a_N = \frac{v^2}{\rho} = \frac{19.9^2}{40} = 9.91$

$a = \sqrt{a_t^2 + a_N^2}$

$= \sqrt{12.62^2 + 9.91^2}$

$= 16.04 \ ft/s^2$

Page 53, problem 12-110: The ball is thrown horizontally with a speed of 8m/s. Find the equation of the path, y=f(x), and then find the balls velocity and the normal and tangential components of acceleration when t=0.25sec.



$V_x = 8\ m/s$

$V_y = -g \cdot t = -9.81t$

$@t=0.25,\quad V_y = -9.81 \cdot 0.25 = 2.4525\ m/s$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{8^2 + 2.4525^2} = 8.37\ m/s$

$V_x = \frac{ds_x}{dt} \qquad V_y = \frac{ds_y}{dt}$

$8 = \frac{ds_x}{dt} \qquad -9.81t = \frac{ds_y}{dt}$

$ds_x = 8 \cdot dt \qquad ds_y = -9.81t \cdot dt$

$s_x = 8 \cdot t \qquad s_y = -4.905t^2$

$x = 8 \cdot t \qquad y = -4.905t^2$

$t = \frac{1}{8} \cdot x \rightarrow y = -4.905 \cdot \left(\frac{1}{8}x\right)^2$

$= -4.905 \cdot \left(\frac{1}{64} \cdot x^2\right)$

$@t = 0.25 \qquad y = -0.076x^2$

$0.25 = \frac{1}{8}x$

$x = 2$

$y = -0.076x^2$

$\dot{y} = -0.153x \qquad @x=2,\ \dot{y} = -0.306$

$\ddot{y} = -0.153$

$$\rho = \frac{\left(1 + (dy/dx)^2\right)^{3/2}}{(d^2y/dx^2)}$$

$$\rho = \frac{\left(1 + (-0.306)^2\right)^{3/2}}{-0.153}$$

$\rho = 7.46$

$a_N = \frac{V^2}{\rho} = \frac{8.37^2}{7.46} = -9.38\ m/s^2$

$g = a = \sqrt{a_N^2 + a_t^2}$

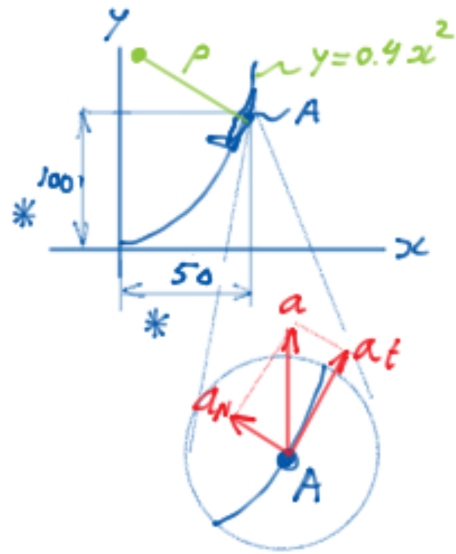
$9.81 = \sqrt{-9.38^2 + a_t^2}$

$96.2361 = 87.9844 + a_t^2$

$a_t^2 = 8.2517$

$a_t = 2.87\ m/s^2$

Page 53, problem 12-111: The plane travels along the vertical parabolic path at a constant speed of 200m/s. Determine the magnitude of acceleration of the plane when it is at point A.



$V=$ constant $\Rightarrow a_t=0$

$y=0.4x^2$

$\dot{y}=0.8x \rightarrow @x=50 \quad , \dot{y}=40$

$\ddot{y}=0.8$

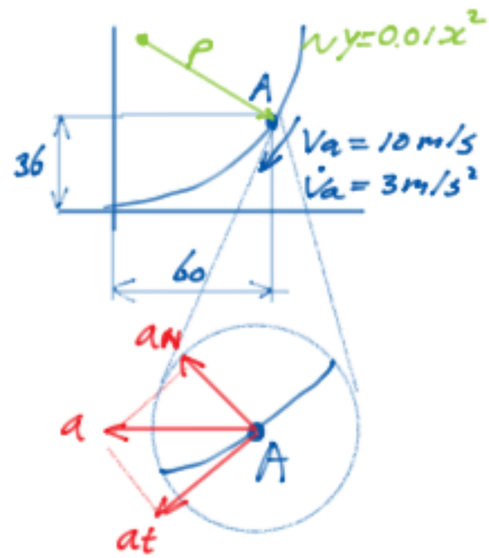
$$\rho=\frac{(1+\dot{y}^2)^{3/2}}{\ddot{y}}$$

$$=\frac{(1+40^2)^{3/2}}{0.8}$$

$$=80075.012 \qquad a_N=\frac{V^2}{\rho}=\frac{200^2}{80075}=0.499 \ m/s^2$$

*Note: in the books drawing x and y coordinates for A are 5km and 10km respectively.

Page 54, 12-113: A toboggan is traveling down along a curve which can be approximated by the parabola y=0.01x^2. Determine the magnitude of its acceleration when it reaches point A, where its speed is Va=10m/s, and it is increasing at the rate of Va'=3m/s^2.



$\dot{V}_a = a_t = 3\ m/s^2$

$V_a = 10\ m/s$

$y = 0.01x^2$

$\dot{y} = 0.02x$, @ $x = 60$, $\dot{y} = 1.2$

$\ddot{y} = 0.02$

$$\rho = \frac{(1+\dot{y}^2)^{3/2}}{\ddot{y}} = \frac{(1+1.44)^{3/2}}{0.02} = 190.57$$

$$a_N = \frac{V^2}{\rho} = \frac{10^2}{190.57} = 0.52\ m/s^2$$

$$a = \sqrt{a_N^2 + a_t^2} = \sqrt{0.52^2 + 3^2} = 3.05\ m/s^2$$