12.1 Rectilinear Kinematics: Continuous Motion

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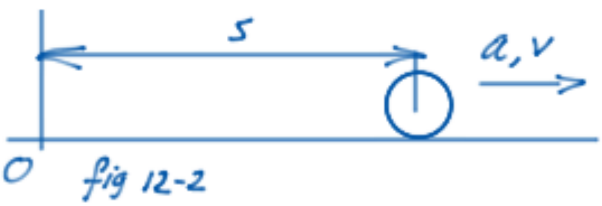
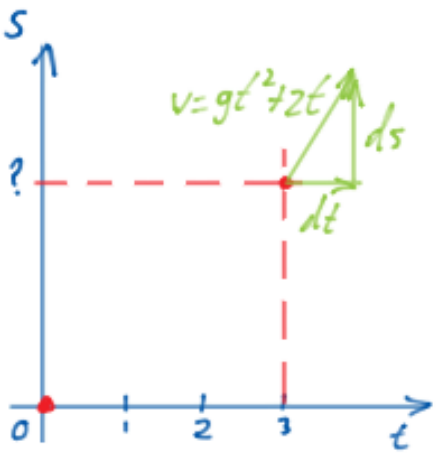
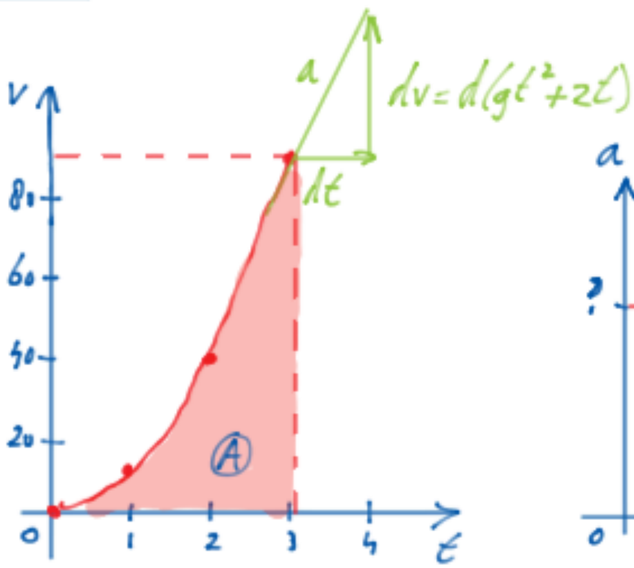
Page 7, example 12-1: The car in fig. 12-2 moves in a straight line such that for a short time it's velocity is defined by v=(9t^2 + 2t) ft/s, where t is in seconds. Determine its position and acceleration when t=3s. When t=0, s=0.

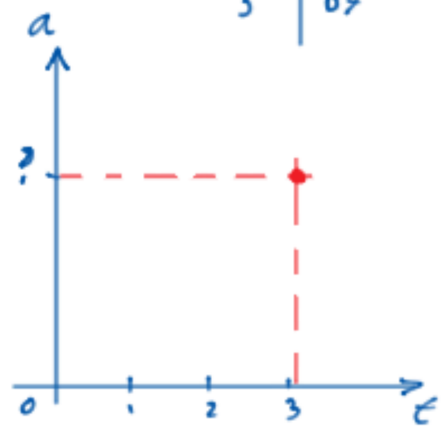


graph →

t	v
0	0
1	11
2	40
3	87







$$\boxed{v = \frac{ds}{dt}}$$

$$9t^2+2t = \frac{ds}{dt}$$

$$ds = (9t^2+2t)\,dt$$

$$s = 3t^3 + t^2 + C$$

$$s_{(0)} = 3\cdot 0^3 + 0^2 + C$$

$$C = 0$$

$$s_{(3)} = 3\cdot 3^3 + 3^2 = 81 + 9 = 90\ ft$$

this equals area A

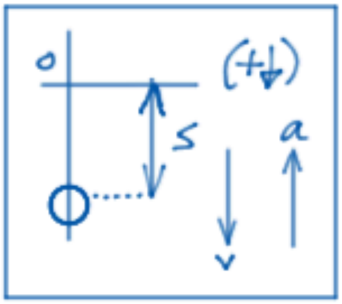
$$\boxed{a = \frac{dv}{dt}} = \frac{d(9t^2+2t)}{dt}$$

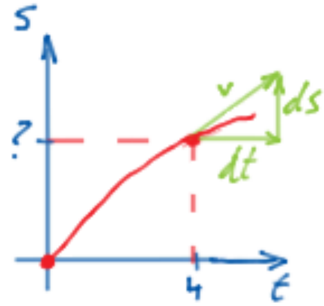
$$= 18t + 2 + C$$

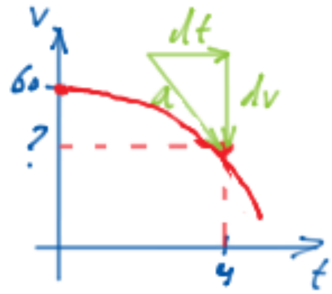
$$a_{(3)} = 18\cdot(3) + 2$$

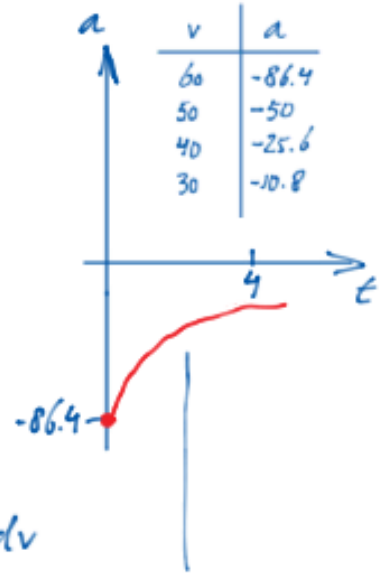
$$= 56\ ft/s^2$$

Page 8, example 12-2: A small particle is fired vertically downward into a fluid medium with an initial velocity of 60m/s. If the projectile experiences a deceleration which is equal to a=(-0.4v^3) m/s^2, where v is measured in m/s, determine the projectile's velocity and position 4s after it is fired.









$\boxed{a = \frac{dv}{dt}}$

$a = -0.4v^3$

$-0.4v^3 = \frac{dv}{dt}$

$-0.4v^3 \cdot dt = dv$

$dt = \frac{dv}{-0.4v^3}$

$dt = \frac{1}{-0.4v^3} \cdot dv$

$dt = -2.5 \cdot \frac{1}{v^3} \cdot dv$

$dt = -2.5 \cdot v^{-3} \cdot dv$

$t = \frac{-2.5}{-2} v^{-2} + c$

$t = 1.25 \cdot v^{-2} + c$

$t = \frac{1.25}{v^2} + c \Rightarrow$

$\boxed{\begin{array}{l} v = 60 \text{ when } t = 0 \\ 0 = \frac{1.25}{60^2} + c \\ c = -0.00347 \end{array}}$

at $t = 4 \Rightarrow 4 = \frac{1.25}{v^2} - 0.00347$

$4.00347 = \frac{1.25}{v^2}$

$v = \sqrt{\frac{1.25}{4.00347}}$

$v = 0.559$ m/s

$\boxed{v = \frac{ds}{dt}}$

$v = \sqrt{\frac{1.25}{t}}$

$\sqrt{\frac{1.25}{t}} = \frac{ds}{dt}$

$ds = \sqrt{\frac{1.25}{t}} \cdot dt$

$ds = \sqrt{1.25} \cdot \sqrt{\frac{1}{t}} \cdot dt$

$ds = 1.12 \cdot \left(\frac{1}{t}\right)^{\frac{1}{2}} \cdot dt$

$ds = 1.12 \cdot ((t)^{-1})^{\frac{1}{2}} \cdot dt$

$ds = 1.12 \cdot (t)^{-\frac{1}{2}} \cdot dt$

$s = \frac{1.12}{0.5}(t)^{0.5} + C$

$s = 2.24(t)^{0.5} + C \Rightarrow \boxed{\left.\begin{array}{l} t = 0 \\ s = 0 \end{array}\right\} c = 0}$

$s_{(4)} = 2.24(4)^{0.5}$

$= 4.48$ m

Page 9, example 12-3: A boy tosses a ball in the vertical direction of the side of a cliff, as shown in fig. 12-4. If the initial velocity of the ball is 15m/s upward, and the ball is released 40m from the bottom of the cliff, determine the maximum height Sb reached by the ball and the speed of the ball just before it hits the ground. During the entire time the ball is in motion, it is subjected to a constant downward acceleration of 9.81m/s^2 due to gravity. Neglect the effect of air resistance.

$(+\uparrow)$

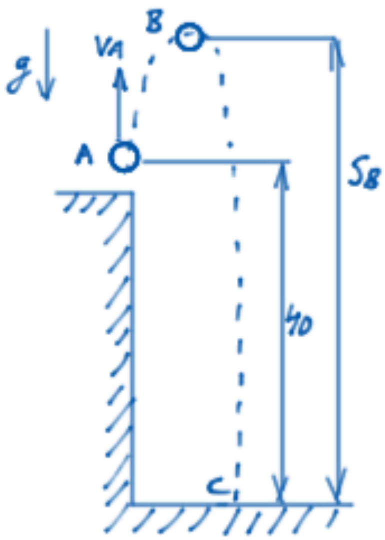
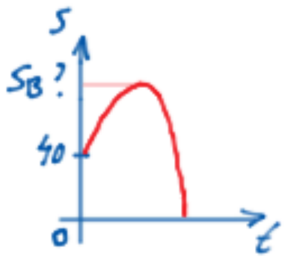
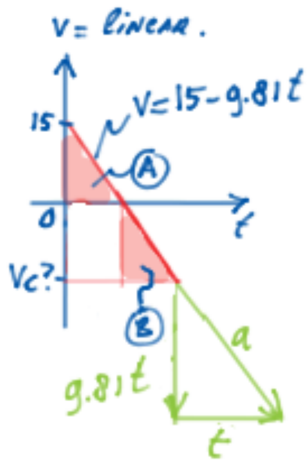
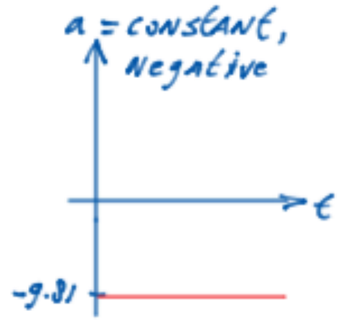


fig 12-4







$@ V=0 \Rightarrow V = 15 - 9.81t$

$0 = 15 - 9.81t$

$-15 = -9.81t$

$t = 1.53\ s.$

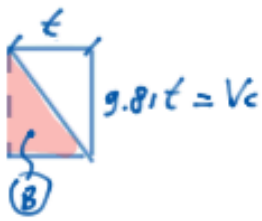
distance traveled up = Ⓐ

$Ⓐ = \frac{15 \cdot 1.53}{2} = 11.47\ m$

max. height $S_B = 40 + 11.47$

$S_B = 51.47\ m$

distance travelled down = Ⓑ = 51.47 m



$\Rightarrow$

$$51.47 = \frac{t \cdot 9.81t}{2} = \frac{9.81t^2}{2} = 4.905t^2$$

$t = 3.24$ sec.

$\Rightarrow$

$V_c(t) = 9.81t$

$V_c(3.24) = 9.81 \cdot 3.24$

$V_c = 31.8\ m/s$

Page 10, example 12-4: A metallic particle is subjected to the influence of a magnetic field such that it travels downward through a fluid that extends from plate A to plate B, fig 12-5. If the particle is released from rest at the midpoint C, s=100mm, and the acceleration is measured as a=(4s)m/s^2, where s is in meters, determine the velocity of the particle when it reaches plate B, s=200mm, and the time it needs to travel van C to B.

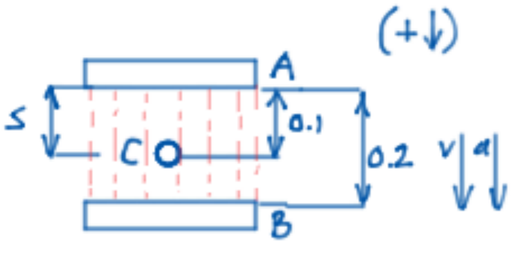
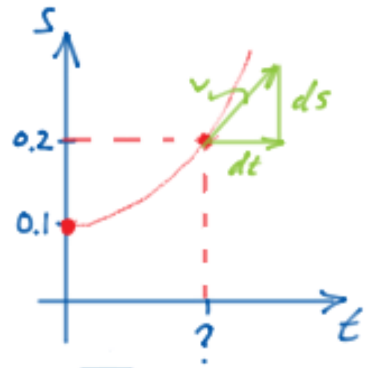
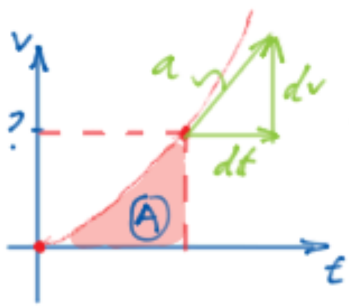


fig 12-5

s	a	v	t
0.1	0.4	0	0
0.2	0.8	?	?
0.3	1.2		



$$\boxed{v = \frac{ds}{dt}} \qquad dt = \frac{ds}{v}$$



$$\boxed{a = \frac{dv}{dt}} \qquad a = 4s$$

$$4s = \frac{dv}{dt} \qquad dt = \frac{ds}{v}$$

$$4s = \frac{dv}{\left(\frac{ds}{v}\right)}$$

$$4s = \frac{v \cdot dv}{ds}$$

$$4s \cdot ds = v \cdot dv$$

$$2s^2 = \frac{1}{2}v^2 + c \quad \rightarrow s = 0.1 \text{ when } v = 0$$

$$2(0.1)^2 = \frac{1}{2}\cdot(0)^2 + c$$

$$0.02 = c \quad \rightarrow s = 0.2,\ v = ?$$

$$2\cdot(0.2)^2 = \frac{1}{2}v^2 + 0.02$$

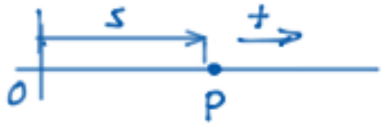
$$0.08 = \frac{1}{2}v^2 + 0.02$$

$$0.06 = \frac{1}{2}v^2$$

$$0.12 = v^2$$

$$v = 0.346 \text{ m/s}$$

Page 11, example 12-5: A particle moves along a horizontal line such that it's velocity is given by v=(3t^2-6t)m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled by the particle during the time interval t=0 to t=3.5, and the particle's average velocity and average speed during this time interval.



$$v = 3t^2 - 6t$$

t	v
0	0
1	-3
2	0
3	9
4	24

$$\boxed{v = \frac{ds}{dt}}$$

$$v = 3t^2 - 6t$$

$$3t^2 - 6t = \frac{ds}{dt}$$

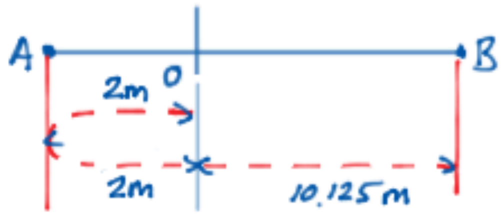
$$ds = (3t^2 - 6t)\,dt$$

$$s = t^3 - 3t^2 + c \qquad @\ t=0,\ s=0 \Rightarrow c=0$$

$$s(t) = t^3 - 3t^2$$

$$A = s(2) - s(0) = (2^3 - 3\cdot 2^2) - (0^3 - 3\cdot 0^2) = -4\ m$$

$$B = s(3.5) - s(2) = (3.5^3 - 3\cdot 3.5^2) - (-4) = 6.125 - (-4) = 10.125\ m$$



total distance traveled is A + B

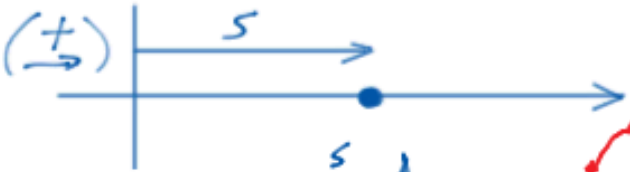
$$A + B = 4m + 10.125m = 14.125m$$

$$\text{average velocity} = \text{total distance traveled / time} = \frac{14.125}{3.5} = 4.036\ m/s$$

$$\text{average speed} = \text{total displacement / time} = \frac{s(3.5) - s(0)}{t(3.5) - t(0)} = \frac{6.125 - 0}{3.5} = 1.75\ m/s$$

Page12, problem 12-1: If a particle has an initial velocity of V0=12ft/sec to right, determine its position when t=10s, if a=2ft/sec^2 to the left. Originally s0=0.

$a = \text{constant} = -2\,ft/sec^2$ $(\overset{+}{\rightarrow})$



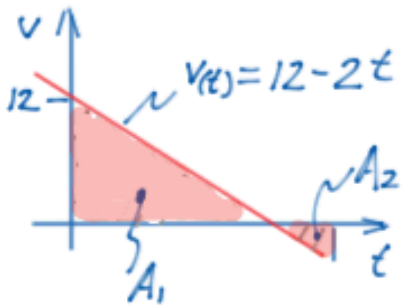
using formula:

$S = S_0 + V_0 \cdot t + \frac{1}{2} a t^2$

$S = 0 + 12 \cdot 10 + \frac{1}{2}(-2) \cdot 10^2$

$S = 0 + 120 + (-100)$

$S = 20\,ft$



graphical solution:
the position of the particle is determined by $A_1 + A_2$

the particle reverses direction when v=0:

$0_{(t)} = 12 + (-2t) \rightarrow -12 = -2t \rightarrow t = 6\,sec$

particle velocity at t=10:

$V_{(10)} = 12 + (-2 \cdot 10) = 12 + (-20) = -8\,ft/sec$

$A_1 = \frac{12 \cdot 6}{2} = 36\,ft$

$A_2 = \frac{-8 \cdot (10-6)}{2} = -16\,ft$

$36 + (-16) = 20\,ft$

calculus solution:

$ds = (12 - 2t)\,dt$

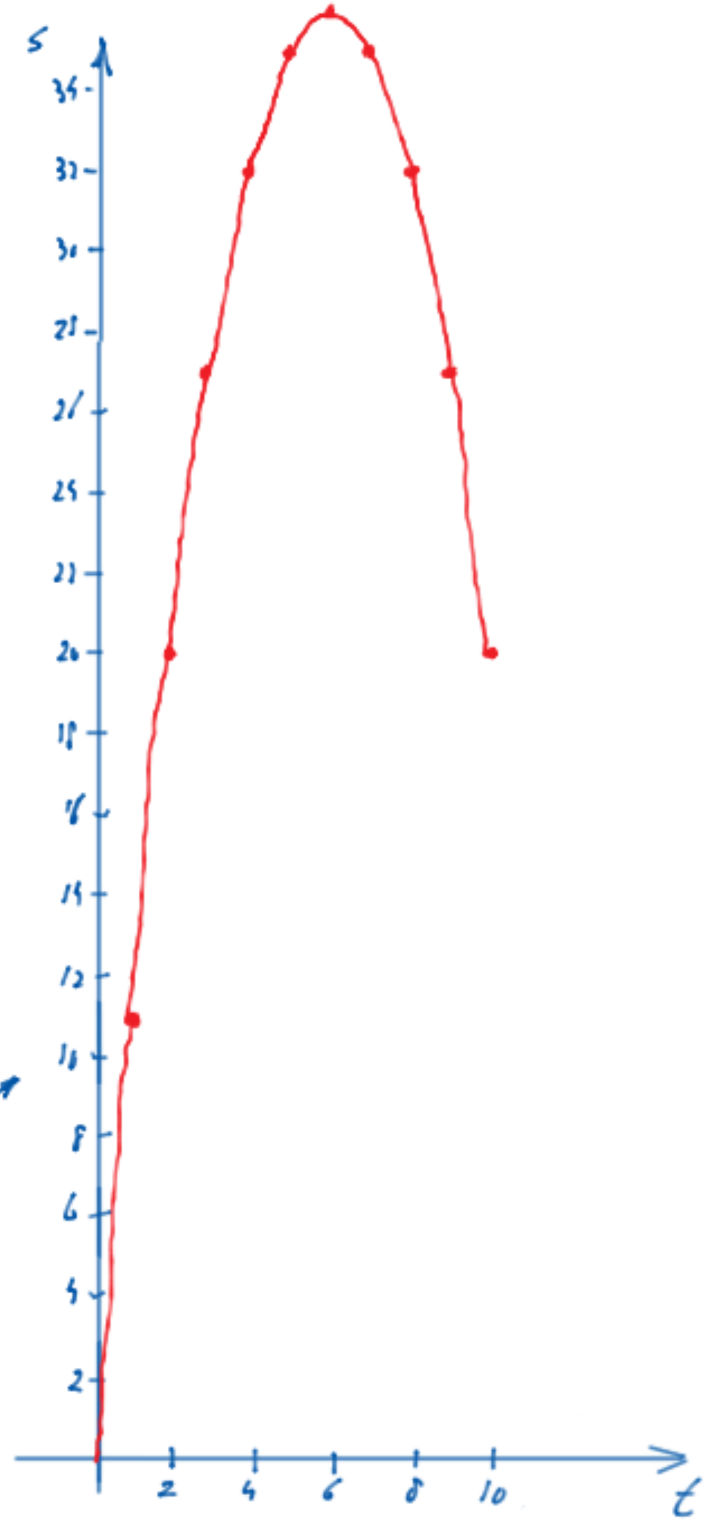
$S = 12t - t^2 + C$

$S_{(10)} = 12 \cdot 10 - 10^2$

$S_{(10)} = 120 - 100 = 20\,ft$

position function is 1st anti-derivative of velocity

t	s
0	0
1	11
2	20
3	27
4	32
5	35
6	36
7	35
8	32
9	27
10	20



Page 12, problem 12-2: From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/sec (55 mi/hr) when it hits the ground? Each floor 12ft higher than the one below it.

$a_c = \text{constant} = 32\ ft/sec^2$

using equations:

$v = v_0 + a_c \cdot t$

$80.7 = 0 + 32 \cdot t$

$80.7 = 32t \rightarrow t = 2.5$ sec.

→ time needed to reach 80.7 ft/sec.

time travelled in 2.5 sec:

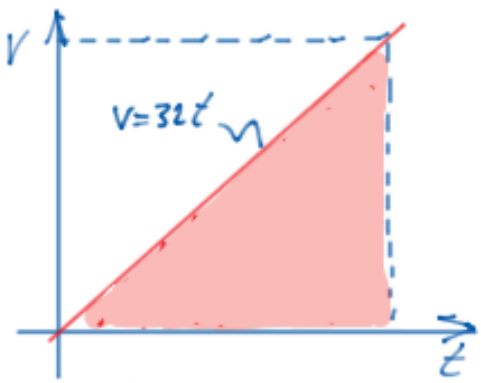
$s = s_0 + v_0 \cdot t + \frac{1}{2} a t^2$

$= 0 + 0 \cdot t + \frac{1}{2} \cdot 32 \cdot 2.5^2$

$= 101.6$ ft

drop from floor $\frac{101.6}{12} = 8.4$

$\Rightarrow$ 9th floor

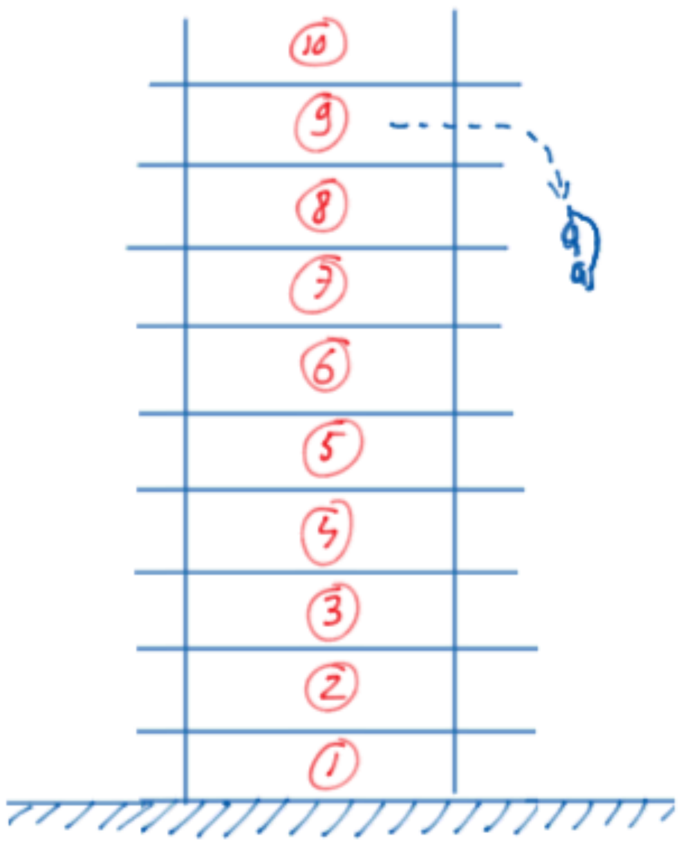


$v = \frac{ds}{dt} = 32t$

$ds = (32t)\,dt$

$s = 16t^2 \rightarrow s_{(2)} = 16 \cdot 2.52^2$

$= \underline{101.7\ ft}$



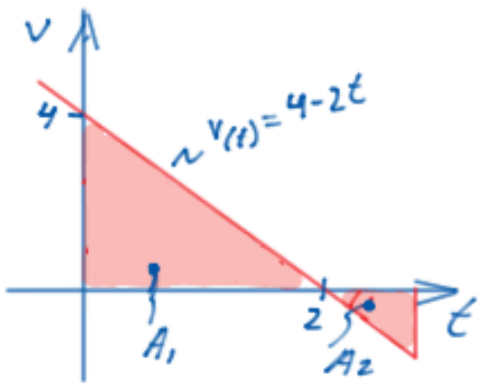
Page 12, problem 12-3: A particle is moving along a straight line such that its position is given by s=(4t-t^2) ft., where t is in seconds. Determine the distance travelled from t=0 to t=5s, the average velocity, and the average speed of the particle during this time interval.

$s(t) = 4t - t^2 \Rightarrow \boxed{v = \frac{ds}{dt}} \Rightarrow v = \frac{d}{dt}(4t - t^2)$

first derivative of pos. $\Rightarrow = 4 - 2t \Rightarrow v(0) = 4 - 2\cdot 0 = 4$ ft/s

$v(5) = 4 - 2\cdot 5 = -6$ ft/s

Average velocity $\Rightarrow v_{avg} = \frac{4+(-6)}{2} = \frac{-2}{2} = -1$ ft/sec



At v=0 the particle reverses direction:

$0(t) = 4-2t$

$-4 = -2t \Rightarrow t = 2$ sec.

$\left.\begin{array}{l} \text{distanse } A_1 = \frac{2\cdot 4}{2} = 4 \text{ ft} \\ \text{distanse } A_2 = \frac{(5-2)\cdot -6}{2} = |9 \text{ ft}| \end{array}\right\} 13 \text{ ft}$

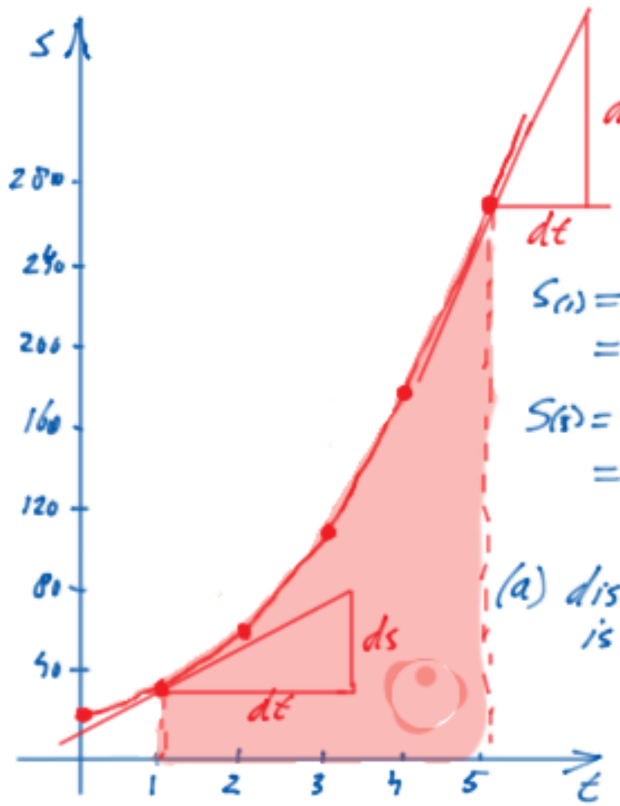
Average speed is $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{13}{5} = 2.6$ ft/sec

Page 12, problem 12-5: A particle is moving along a straight line path such that it's position is defined by s=(10t^2+20)mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from t=1 s to t=5 s, (b) the average velocity of the particle during this time interval, and (c) the acceleration at t=1 s.

$s = 10t^2 + 20$

t	s
0	20
1	30
2	60
3	110
4	180
5	270



$$S_{(1)} = 10t^2 + 20 = 10 \cdot 1^2 + 20 = 30$$

$$S_{(5)} = 10 \cdot t^2 + 20 = 10 \cdot 5^2 + 20 = 270$$

(a) distance travelled is 270−30= 240 mm

(b) Average velocity is:

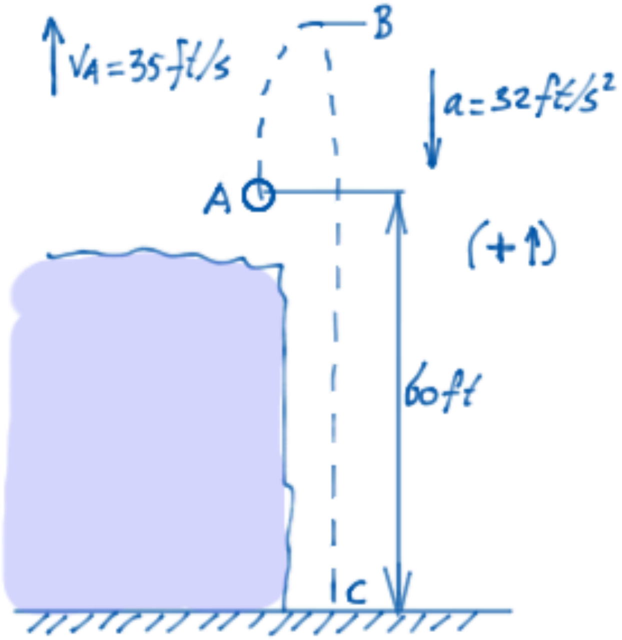
$$\boxed{v = \frac{ds}{dt}} = \frac{d}{dt}(10t^2+20) \rightarrow 20t$$

$$\left.\begin{array}{l} V_{(1)} = 20 \cdot 1 = 20 \text{ mm/s} \\ V_{(5)} = 20 \cdot 5 = 100 \text{ mm/s} \end{array}\right\} V_{Avg} = \frac{100+20}{2} = 60 \text{ mm/s}$$

$$\boxed{a = \frac{dv}{dt}} = \frac{d}{dt}(20t) \rightarrow 20$$

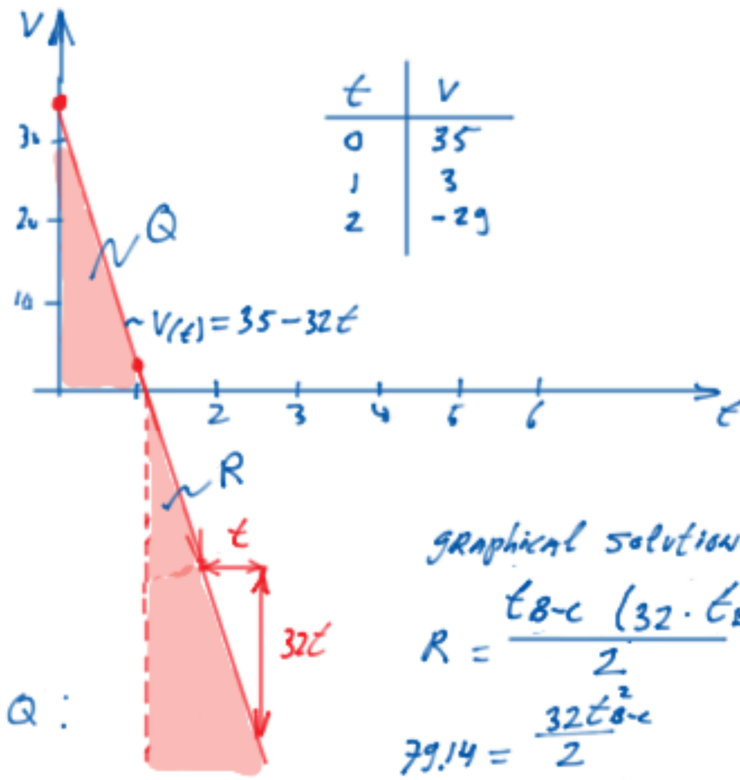
(c) acceleration is constant $a_{(1)} = 20 \text{ mm/s}^2$

Page 12, problem 12-6: A ball is thrown vertically upward from the top of a ledge with an initial velocity of Va=35ft/sec. Determine (a) how high above the top of the cliff the ball will go before its stops at B, (b) the time Ta-b it takes to reach its maximum height, and (c) the total time Ta-c needed for it to reach the ground at C from the instant it is released.



gravity will cause the ball to reduce speed at a constant rate that can be described by function:

$V(t) = 35 - a \cdot t = 35 - 32t$



t	V
0	35
1	3
2	-29

the ball will reach B when V=0:

$0 = 35 - 32t$

$-35 = -32t \rightarrow$ (b) $t_{A-B} = 1.09$ sec.

the distance travelled is equal to area Q:

$\frac{1.09 \times 35}{2} = 19.14$ ft

graphical solution:

$R = \frac{t_{B-C}(32 \cdot t_{B-C})}{2}$

$79.14 = \frac{32 t_{B-C}^2}{2}$

$158.28 = 32 t_{B-C}^2$

$\boxed{t_{B-C} = 2.2 \text{ sec}}$

using equation:

distance B-C = 60 + 19.14 ft = 79.14 ft → equal to area R

$s = \frac{1}{2} a t^2$

$79.14 = \frac{1}{2} \cdot 32 \cdot t^2$

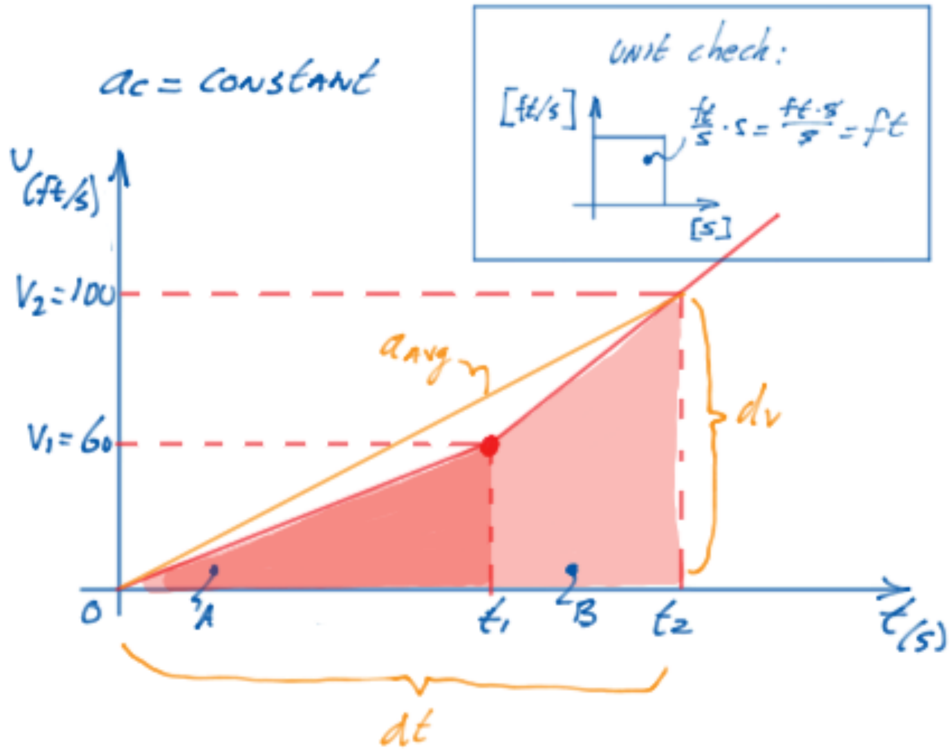
$79.14 = 16 t^2 \rightarrow t_{B-C} = 2.2$ sec

(c) $t_{A-B} = 1.09$

$t_{B-C} = 2.2 \; +$

$t_{A-C} = 3.29$ sec

Page 12, problem 12-7: A car, initially at rest, moves along a straight road with constant acceleration such that it attains a velocity of V=60ft/s when s=150ft. Then after being subjected to another constant acceleration, it attains a final velocity of V=100ft/s when s=325ft. Determine the average velocity and average acceleration of the car for the entire 325ft displacement.



graphical solution:

initial distance travelled is 150 ft = area A:

$$A = \frac{v_1 \cdot t_1}{2}$$

$$150 = \frac{60 \cdot t_1}{2}$$

$$300 = 60 \cdot t_1$$

$$t_1 = 5 \text{ sec.}$$

⇓

$A + B = 325\ ft$ = total distance travelled

$150 + B = 325\ ft$

$B = 175\ ft$

⇒

$$B = 175 = 60 \cdot x + \tfrac{1}{2} \cdot 40 \cdot x$$

$$175 = 60x + 20x$$

$$175 = 80x$$

$$x = 2.1875$$

⇐

$$t_2 = t_1 + x$$

$$= 5 + 2.1875$$

$$= 7.1875 \text{ sec.}$$

(a)* average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{325}{7.1875} = 45.2\ ft/s$

(b) $a_{avg} = \dfrac{dv}{dt} = \dfrac{100}{7.1875} = 13.9\ ft/sec^2$

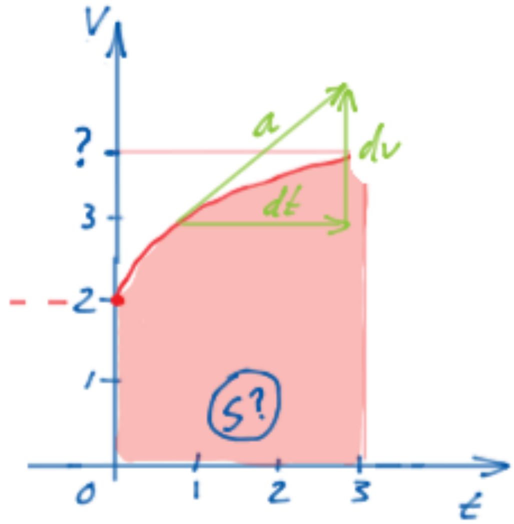
* note that the average velocity is asked, while the average speed is given as answer on page 571

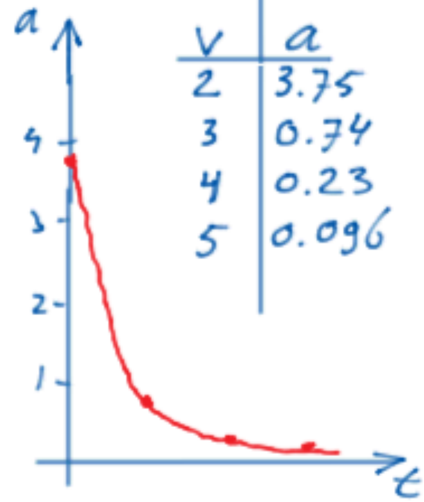
Page 12, problem 12-9: When a train is travelling along a straight track at 2m/s, it begins to accelerate at a=(60V^-4)m/s^2, where V is in m/s. Determine the velocity V and the position of the train 3sec. after the acceleration.

$a_{(v)} = 60v^{-4}$

$a =$ Not constant

$a = 60v^{-4}\ m/s^2 \rightarrow a$ is a function of $v \rightarrow a_{(v)} = 60 \cdot \frac{1}{v^4}$





v	a
2	3.75
3	0.74
4	0.23
5	0.096

$$\left.\begin{array}{l}\boxed{a = \frac{dv}{dt}} \\ a = 60v^{-4}\end{array}\right\} \quad 60v^{-4} = \frac{dv}{dt}$$

$$60v^{-4} \cdot dt = dv$$

$$dt = \frac{dv}{60v^{-4}}$$

$$dt = \frac{1}{60v^{-4}} \cdot dv$$

$$dt = \frac{1}{60} v^4 \cdot dv$$

$$t = \frac{1/60}{5} v^5 + c$$

$$t = 0.003v^5 + c$$

$$\left.\begin{array}{l} t=0 \\ v=2 \end{array}\right\} \quad 0 = 0.003 \cdot 2^5 + c$$

$$0 = 0.096 + c$$

$$c = -0.096$$

$$t=3 \} \quad 3 = 0.003 \cdot v^5 - 0.096$$

$$3.096 = 0.003v^5$$

$$v^5 = 1032$$

$$v = 4\ m/s$$

Calculate S $\Rightarrow$

$$v^5 = \frac{t}{0.003}$$

$$v^5 = 333t$$

$$v = \sqrt[5]{333t}$$

$$\left.\boxed{v = \frac{ds}{dt}}\right\} \quad \sqrt[5]{333t} = \frac{ds}{dt}$$

$$ds = \sqrt[5]{333t} \cdot dt$$

$$ds = \sqrt[5]{333} \cdot \sqrt[5]{t} \cdot dt$$

$$ds = 3.2t^{\frac{1}{5}} \cdot dt$$

$$s = 2.67t^{1.2} + c$$

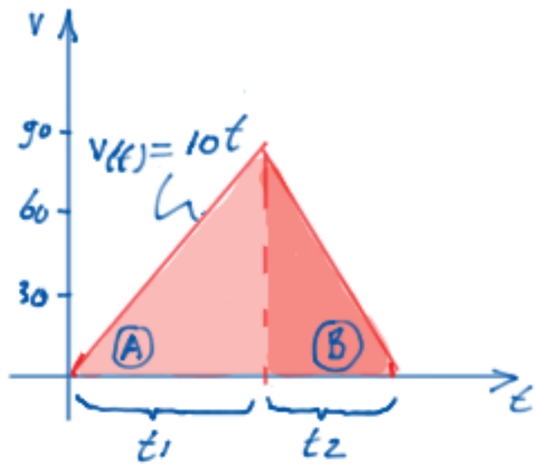
$s=0$ @ $t=0$; $\Rightarrow c=0$

$$s_{(3)} = 2.67 \cdot 3^{1.2} = 9.98\ m$$

Page 12, problem 12-10: A race car uniformly accelerates at 10ft/s^2 from rest, reaches a maximum speed of 60mi/h, and then decelerates uniformly to a stop. Determine the total elapsed time if the distance travelled was 1500ft.

$$V_{max} = 60 \text{ miles/hour}$$
$$= \frac{60}{3600} \text{ miles/sec}$$
$$= \frac{60}{3600} \cdot 5280 \text{ ft/sec}$$
$$\simeq 88 \text{ ft/sec}$$

$a = \text{constant} = 10 ft/s^2$



$$V(t) = 10t$$
$$88 = 10t$$
$$t_1 = 8.8 \text{ sec.}$$

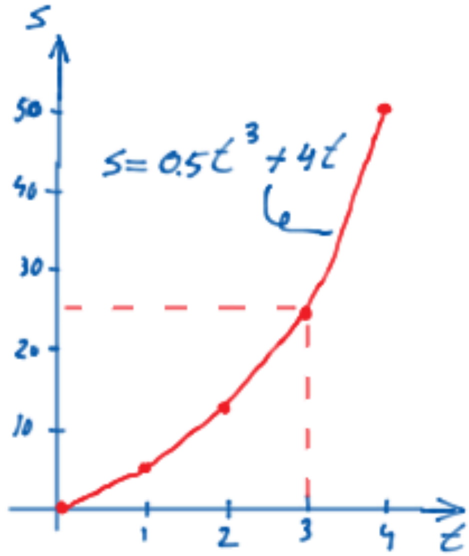
$$A = \frac{88 \times 8.8}{2} = 387.2 \text{ ft}$$

$$A + B = 1500 \text{ ft}$$
$$387.2 + B = 1500$$
$$B = 1112.8 \text{ ft}$$

$$B = \frac{88 \times t}{2} = 1112.8$$
$$88 \times t = 2225.6$$
$$t_2 = 25.29 \text{ sec.}$$

$$t_{total} = 8.8 + 25.29$$
$$= 34 \text{ sec}$$

Page 13, problem 12-11: A small metal particle passes through a fluid medium under the influence of magnetic attraction. The position of the particle is defined by s=(0.5t^3+4t)inch., where t is in seconds. Determine the position, velocity, and acceleration of the particle when t=3s.



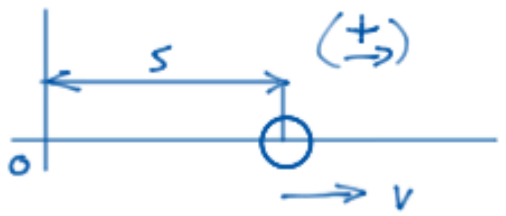
t	s
0	0
1	4.5
2	12
3	25.5
4	50

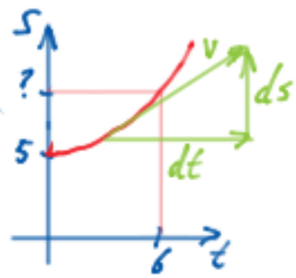
$$S_{(3)} = 0.5 \cdot 3^3 + 4 \cdot 3$$
$$= 13.5 + 12$$
$$= 25.5 \text{ inch.}$$

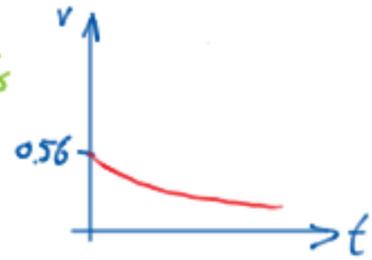
$$\boxed{v = \frac{ds}{dt}} = \frac{d}{dt}(0.5t^3 + 4t)$$
$$v = 1.5t^2 + 4$$
$$V_{(3)} = 1.5 \cdot 3^2 + 4 = 17.5 \text{ inch/sec}$$

$$\boxed{a = \frac{dv}{dt}} = \frac{d}{dt}(1.5t^2 + 4)$$
$$a = 3t$$
$$a_{(3)} = 3 \cdot 3 = 9 \text{ inch/sec}^2$$

Page 13, problem 12-13: A particle travels to the right along a straight path with a velocity v=[5/(4+s)]m/s, where s is in meters. Determine its position when t=6s if s=5m when t=0.







t	s	v
0	5	0.56
	6	0.5
	7	0.45
	8	0.42
	9	0.38
	10	0.36

$$\left.\begin{array}{l}\boxed{v=\dfrac{ds}{dt}} \\ v=\dfrac{5}{4+s}\end{array}\right\} \quad \frac{ds}{dt}=\frac{5}{4+s}$$

$$ds=\left(\frac{5}{4+s}\right)\cdot dt$$

$$dt=\frac{ds}{(5/(4+s))}$$

$$dt=\frac{(4+s)ds}{5}$$

$$dt=\frac{4ds+sds}{5}$$

$$dt=\frac{1}{5}(4ds+s\cdot ds)$$

$$dt=0.8ds+\frac{1}{5}s\cdot ds$$

$$t=0.8s+0.1s^2+C$$

@$t=0, s=5$: $\quad 0=0.8\cdot(5)+0.1(5)^2+C$

$$0=4+2.5+C \Rightarrow C=-6.5$$

$t=6, C=-6.5$: $\quad 6=0.8s+0.1s^2-6.5$

$$60=8s+s^2-65$$

$$s^2+8s-125=0$$

Quadratic formula: page 555

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$s=\frac{-8\pm\sqrt{8^2-4\cdot1\cdot-125}}{2\cdot1}$$

$$s=\frac{-8\pm23.75}{2} \Rightarrow \begin{array}{l} s=7.87\text{ m} \\ s=-15.88\text{ m}\end{array}$$

⇓

'ball park' calculation assuming average speed over 5 meters:

$$V_{avg}=\frac{0.56+0.36}{2}=0.46\text{ m/s}$$

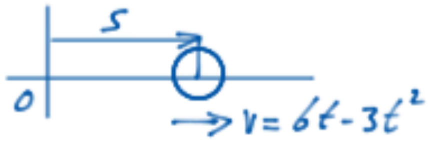
distance traveled in 6 sec:

$$0.46\cdot6=2.76\text{ m}$$

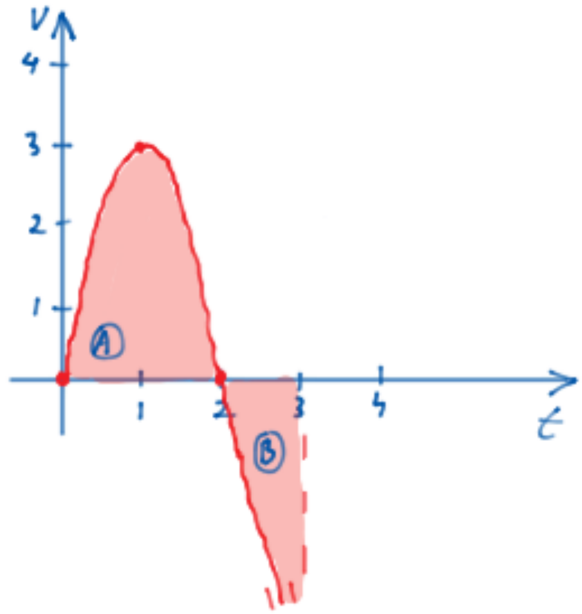
$$s_{(0)}=\underline{\;5\text{ m}\;}$$

$$s_{(6)}\approx7.76\text{ m}$$

Page 13, problem 12-14: The velocity of a particle traveling along a straight line is v=(6t-3t^2)m/s, where t is in seconds. If s=0 when t=0, determine the particle's deceleration and position when t=3s. How far has the particle traveled during the 3-s time interval, and what is the average speed?



t	v
0	0
1	3
2	0
3	-9



Position:

$$v = \frac{ds}{dt} = 6t - 3t^2$$

$$ds = (6t - 3t^2)\,dt$$

$$s = 3t^2 - t^3$$

$$s_{(3)} = 3 \cdot 3^2 - 3^3$$

$$= 27 - 27 = 0\ m$$

deceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}(6t - 3t^2)$$

$$a = 6 - 6t$$

$$a_{(3)} = 6 - 6 \cdot 3$$

$$= -12\ m/s^2$$

distance traveled is Area A + B

when $v=0$: $v = 6t - 3t^2$

$$0 = 6t - 3t^2$$

$$0 = (t + 0) \cdot (-3t + 6)$$

$$t = 0$$

$$-3t = 6 \Rightarrow t = 2$$

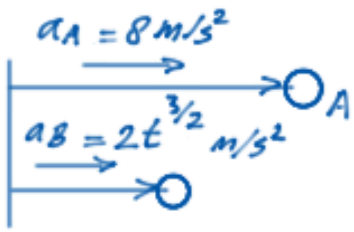
Area Ⓐ: $\int_0^2 (6t - 3t^2)\,dt \rightarrow \Big|_0^2 3t^2 - t^3 \rightarrow (3 \cdot 2^2 - 2^3) - (0) = 4m$

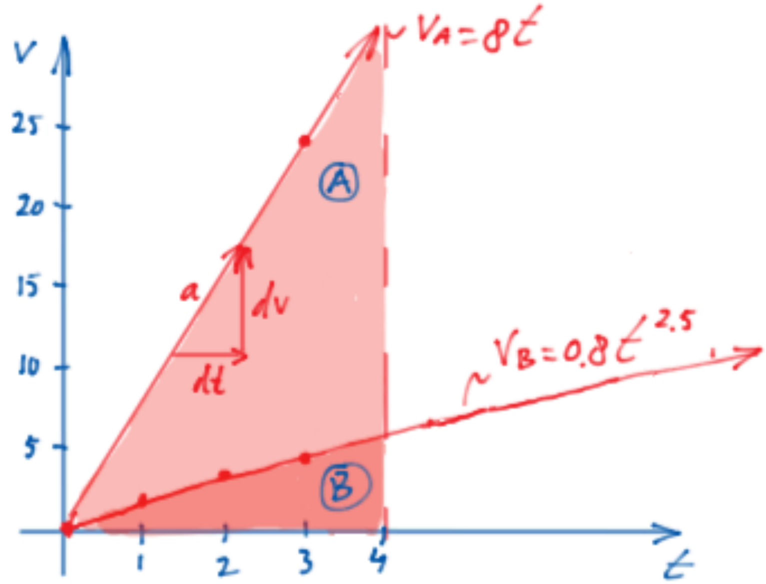
Area Ⓑ: $\int_2^3 (6t - 3t^2)\,dt \rightarrow \Big|_2^3 3t^2 - t^3 \rightarrow (3 \cdot 3^2 - 3^3) - (3 \cdot 2^2 - 2^3) = (0) - (4) = -4m$

total distance traveled $= 4 + |-4| = 8m$

average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{8}{3} = 2.7\ m/s$

Page 13, problem 12-17: At the same instant, two cars A and B start from rest at a stop line. Car A has a constant acceleration of aA=8m/s^2, while car B has an acceleration of aB=(2t(3/2))m/s^2, where t is in seconds. Determine the distance between the cars when A reaches a velocity of Va=120km/h.





$$V_A = \frac{120\,km/h}{3600\,sec} = 0.033 \times 1000 = 33.33\,m/s$$

CAR A:

$a_A = 8\,m/s^2$, $\boxed{a_A = \frac{dv}{dt}}$ $\quad \frac{dv}{dt} = 8$

$dv = 8\,dt$

@t=0, v=0 ⇒ C=0: $v = 8t + C \rightarrow \boxed{v = \frac{ds}{dt}} \rightarrow \frac{ds}{dt} = 8t$

@t=?, v=33.33: $\quad 33.33 = 8t$

$t = 4.17$ sec

$ds = 8t \cdot dt$

$s = 4t^2 + C$: @t=0, s=0 ⇒ C=0

Position CAR A: $S_{(4.17)} = 4 \cdot 4.17^2 = 69.56\,m$

Position CAR B: $S_{(4.17)} = 0.23 \cdot (4.17)^{3.5} = 34.06\,m$ −

$\Delta_{A-B} = 35.5\,m$

CAR B:

$a_B = 2t^{1.5}\,m/s^2$, $\boxed{a_B = \frac{dv}{dt}}$ $\quad \frac{dv}{dt} = 2t^{1.5}$

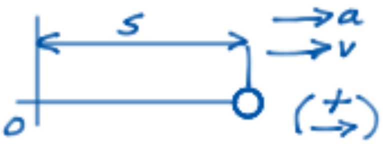
$dv = 2t^{1.5} \cdot dt$

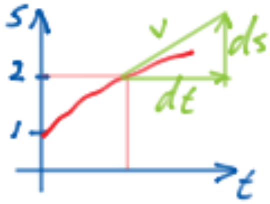
$v = 0.8t^{2.5} + C$, $\boxed{v = \frac{ds}{dt}}$ $\quad \frac{ds}{dt} = 0.8t^{2.5}$

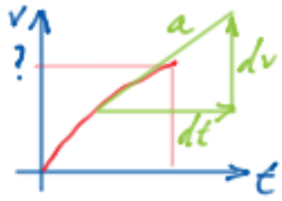
$ds = 0.8t^{2.5} \cdot dt$

@t=0, s=0 ⇒ C=0: $s = 0.23t^{3.5} + C$

Page 13, problem 12-18: A particle moves along a straight path with an acceleration of a=(5/s)m/s^2, where s is in meters. Determine the particle's velocity when s=2m if it is released from rest when s=1m.







s	a
1	5
2	2.5

$$\left.\begin{array}{l} a = \frac{5}{s}\, m/s^2 \\ a = \frac{dv}{dt} \end{array}\right\} \frac{dv}{dt} = \frac{5}{s}$$

$$\left.\begin{array}{l} dv = \frac{5}{s} \cdot dt \\ v = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{v} \end{array}\right\} \begin{array}{l} dv = \frac{5}{s} \cdot \frac{ds}{v} \\ dv = 5 \cdot \frac{1}{s} \cdot \frac{1}{v} \cdot ds \end{array}$$

$$v \cdot dv = 5 \cdot \frac{1}{s} \cdot ds$$

$$v \cdot dv = \frac{5\, ds}{s}$$

$$v \cdot dv = 5 \cdot \frac{ds}{0 + 1s} \quad - \quad \begin{array}{l} a = 0 \\ b = 1 \end{array}$$

$$\frac{1}{2} v^2 = 5 \cdot \ln(s) + C$$

$@\, s = 1,\ v = 0 \Rightarrow C = 0$

$@\, s = 2,\ C = 0 \Rightarrow$

$$\frac{1}{2} v^2 = 5 \cdot \ln(2)$$

$$\frac{1}{2} v^2 = 3.47$$

$$v^2 = 6.93$$

$$v = 2.63 \text{ m/s}$$

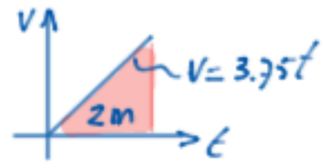
PAGE 556

$$\boxed{\int \frac{dx}{a + bx} = \frac{1}{b} \ln(a + bx) + C}$$

APPROXIMATION

$\Downarrow$

$$a_{Avg} = \frac{5 + 2.5}{2} = 3.75$$



$$2 = \frac{t \cdot 3.75t}{2}$$

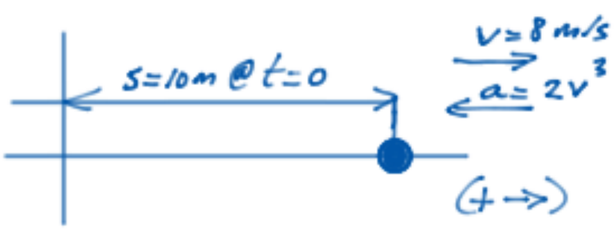
$$2 = 1.875\, t^2$$

$$t = 1.03$$

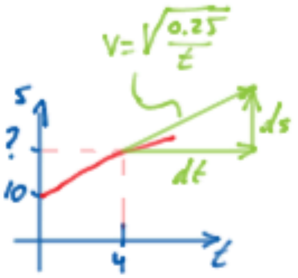
$$V(1.03) = 3.75 \cdot 1.03$$

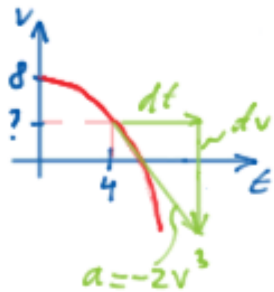
$v\,@\, s=2:\ v \approx 3.87$ m/s

Page 13, problem 12-21: A particle moving along a straight line is subjected to a deceleration a=(-2v^3)m/s^2, where v is in m/s. If it has a velocity v=8m/s and a position s=10m when t=0, determine its velocity and position when t=4s.



t	s	v	a
0	10	8	-1024
4	?	?	





$$\boxed{v = \frac{ds}{dt}} \qquad v = \sqrt{\frac{0.25}{t}}$$

$$\sqrt{\frac{0.25}{t}} = \frac{ds}{dt}$$

$$ds = \sqrt{\frac{0.25}{t}}\, dt$$

$$ds = \sqrt{0.25} \cdot \sqrt{\frac{1}{t}} \cdot dt$$

$$ds = 0.5\, t^{-0.5} \cdot dt$$

$$s = t^{0.5} + C$$

$$\Downarrow$$

$@t=0,\ s=10$

$10 = 0 + C \Rightarrow C = 10$

$@t=4 \Rightarrow s = 4^{0.5} + 10 = 12\text{ m}$

$$\boxed{a = \frac{dv}{dt}} \qquad a = -2v^3$$

$$-2v^3 = \frac{dv}{dt}$$

$$-2v^3 \cdot dt = dv$$

$$dt = \frac{dv}{-2v^3}$$

$$dt = -\frac{1}{2} \cdot \frac{1}{v^3} \cdot dv$$

$$dt = -\frac{1}{2} \cdot v^{-3} \cdot dv$$

$$t = 0.25\, v^{-2} + C$$

$$v = \sqrt{\frac{0.25}{t}} \leftarrow t = \frac{0.25}{v^2} + C \rightarrow @t=0,\ v=8 \rightarrow 0 = \frac{0.25}{8^2} + C$$

$$C = -0.0039$$

$$@t=4 \rightarrow 4 = \frac{0.25}{v^2} - 0.0039$$

$v = 0.248\text{ m/s}$

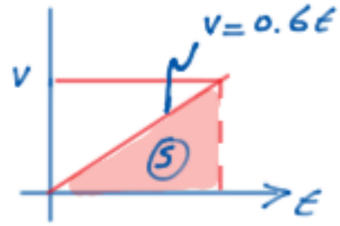
calculation by approximation:

$a = 4s \ m/s^2$

s	a	v	t
0.1	0.4	0	0
0.2	0.8	?	?
0.3	1.2		

$a_{avg} = \frac{0.4+0.8}{2} = 0.6 \ m/s^2$

$s = 0.2 - 0.1 = 0.1 \ m$



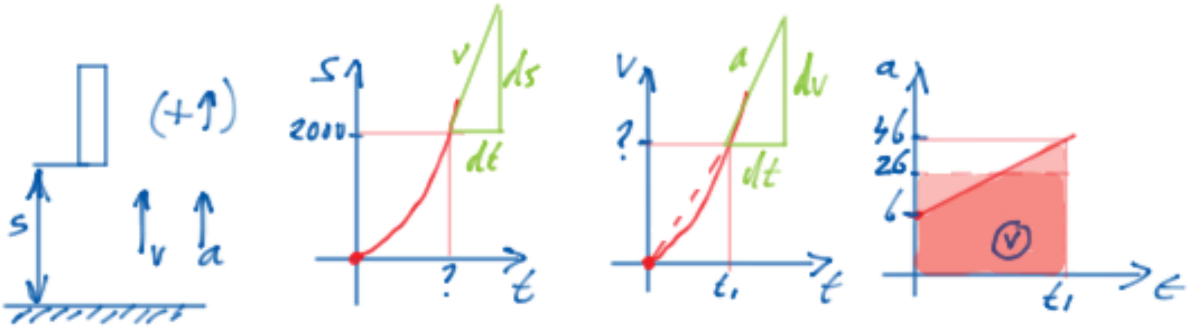
$s = \frac{0.6t \cdot t}{2} \Leftarrow \boxed{s = \frac{1}{2}at^2}$

$0.1 = 0.3t^2$

$t = \boxed{0.58 \ sec}$

$V = 0.6t = 0.6 \cdot 0.58 = \boxed{0.346 \ m/s}$

Page 14, problem 12-22: The acceleration of a rocket travelling upward is given by a=(6+0.02s) m/s^2, where s in meters. Determine the rocket's velocity when s=2km and the time needed to reach this elevation. Initially, v=0 and s=0 when t=0.



t	s	v	a
0	0	0	6
?	2000	?	46

graphical solution:

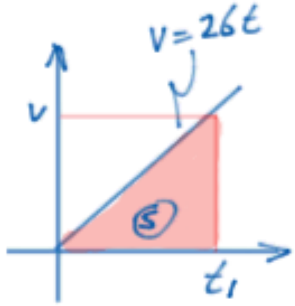
a is linear increasing from 6 to 46 m/s^2

the speed at $t=t_1$ equals the area (v) under graph a-t.

the same area can be defined using a constant acceleration a_c

$$a_c = \frac{6+46}{2} = 26\ m/s^2$$

this gives a linear increasing velocity ⇒ where area (s) under the graph equals the distance traveled.



$$(v) = 26 t_1 = 26 \cdot 12.4 = \underline{322.5\ m/s}$$

⇑

$$(s) = \frac{t_1 \, 26 t_1}{2}$$

⇒ $2000 = 13 t_1^2$

$$\underline{t_1 = 12.4\ sec.}$$

calculus solution:

$$\boxed{a = \frac{dv}{dt}} \qquad a = 6+0.02s$$

$$6+0.02s = \frac{dv}{dt}$$

$$\boxed{v = \frac{ds}{dt}} \rightarrow dt = \frac{ds}{v}$$

$$6+0.02s = \frac{dv}{(ds/v)}$$

$$6+0.02s = \frac{v \cdot dv}{ds}$$

$$(6+0.02s)\,ds = v \cdot dv$$

$$\frac{1}{2}v^2 = 6s + 0.01s^2 + C \; : \; v=0,\ s=0 \Rightarrow C=0$$

$$s=2000 : \; \frac{1}{2}v^2 = 6 \cdot 2000 + 0.01 \cdot 2000^2$$

$$v^2 = 104000$$

$$v = 322.5\ m/s$$

$$\frac{1}{2}v^2 = 6s + 0.01s^2$$

$$v^2 = 12s + 0.02s^2$$

$$v = \sqrt{12s + 0.02s^2}$$

$$\boxed{v = \frac{ds}{dt}}$$

$$\sqrt{12s + 0.02s^2} = \frac{ds}{dt}$$

$$dt = \frac{ds}{\sqrt{12s + 0.02s^2}}$$

$$t = \frac{1}{\sqrt{0.02}} \ln\left(\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right) + C$$

when $t=0$, $s=0$: $0 = 7.07 \ln(42.43) + C$

$$0 = 26.5 + C \Rightarrow C = -26.5$$

when $s = 2000$: $t = 7.07 \ln(322.5 + 282.84 + 42.43) - 26.5$

$$t = (7.07 \cdot 6.47) - 26.5$$

$$= 19.26 \text{ sec}$$

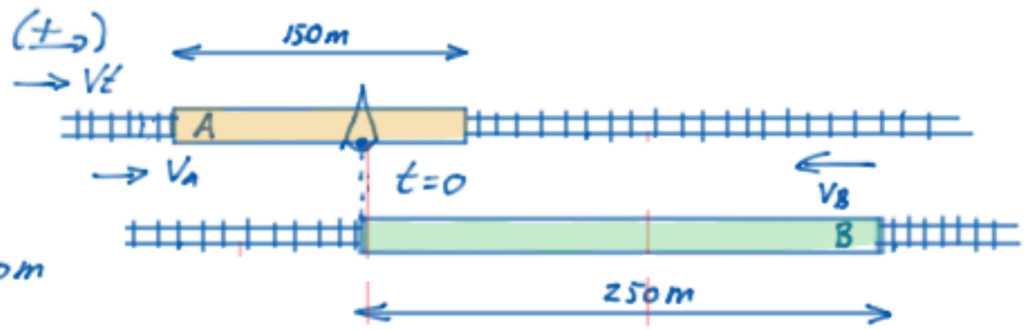
Page 557, integrals

$$\int \frac{dx}{\sqrt{a + bx + cx^2}} = \frac{1}{\sqrt{c}} \ln\left(\sqrt{a + bx + cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right) + C \qquad \text{for } c > 0$$

Page 14, problem 12-23: Two trains are traveling in opposite directions on parallel tracks. Train A is 150m long and has a speed which is twice as fast as train B, which is 250m long. Determine the speed of each train if a passenger in train A observes that train B passes in 4s.

$a = 0$

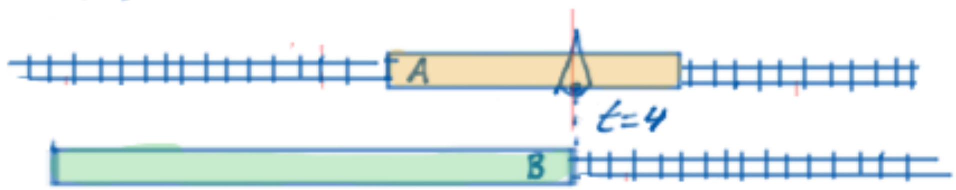
$v = \text{constant}$



Assuming train A is not moving, train B, which is 250m long passes in 4 seconds.

So the speed is $\frac{250}{4} = 62.5\ m/s$

We can state that the total speed (Vt) is 62.5 m/s.



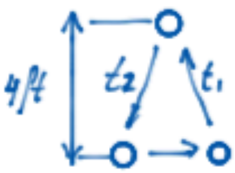
$$\left.\begin{array}{l} V_t = V_A + V_B \\ V_A = 2V_B \end{array}\right\} V_t = 2V_B + V_B$$

$$V_t = 3V_B$$

$$V_B = \frac{V_t}{3} = \frac{62.5}{3} = 20.8\ m/s$$

$$V_A = 2 \cdot V_B = 2 \cdot 20.8 = 41.7\ m/s$$

Page 14, problem 12-25: The juggler maintains the motion of three balls, such that each rises to a height of 4ft. If two balls are in the air at any one time, determine the time the third ball must remain in her hand after the first ball is thrown.



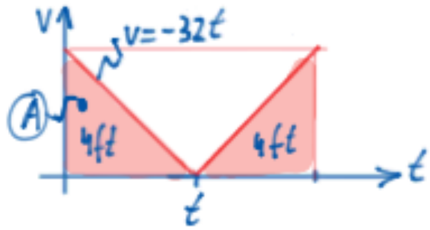
$g = 32\ ft/s^2$

$\downarrow a\ (=g)$

$(+\uparrow)$

The time available is equal to the 'air' time of ball one.

Below graph shows the velocity vs time of ball one.



$Ⓐ = 4ft = \frac{t \cdot 32t}{2}$

$4 = 16t^2$

$t = 0.5\ sec \Rightarrow$ time to reach 4ft

time available is $0.5 \cdot 2 = 1\ sec.$

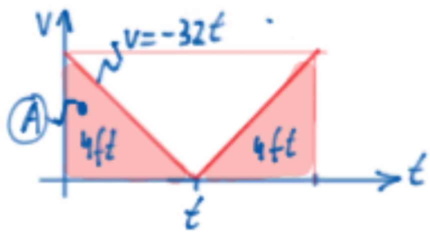
Page 14, problem 12-26: The juggler throws a ball into the air 4ft above her hand. How much time will elapse before she must catch it at the same elevation from which she threw it? What would be the elapsed time if she threw it 8 ft into the air?

$g = 32\ ft/s^2$

$\downarrow a\ (=g)$

$(+\uparrow)$

Below graph shows the velocity vs time.



$$Ⓐ = 4ft = \frac{t \cdot 32t}{2}$$

$$4 = 16t^2$$

$t = 0.5$ sec $\Rightarrow$ time to reach 4ft

total time is $0.5 \cdot 2 = 1$ sec.

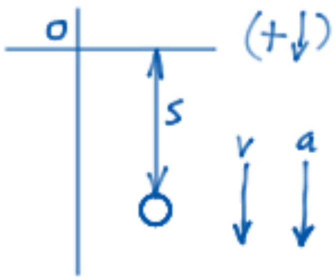
when Ⓐ = 8ft $\Rightarrow$ $8ft = \frac{t \cdot 32}{2}$

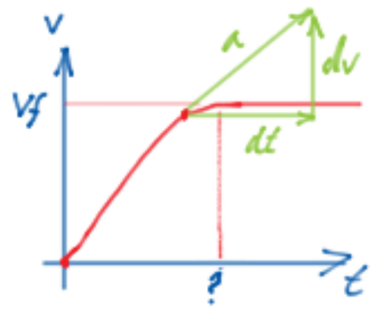
$$8 = 16t^2$$

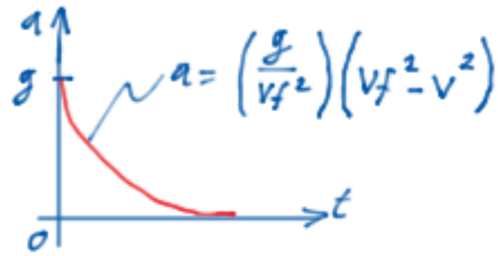
$$t = 0.707$$

total time is $0.707 \cdot 2 = 1.41$ sec.

Page 14, problem 12-29: When a particle falls through the air, it's initial acceleration a=g diminishes until it is zero, and thereafter it falls at a constant velocity Vf. If this variation of the acceleration can expressed as a=(g/Vf^2)(Vf^2-V^2), determine the time needed for the velocity to become V<Vf. Initially the particle falls from rest.







$$\left.\begin{aligned} a &= \frac{dv}{dt} \\ a &= \left(\frac{g}{Vf^2}\right)\cdot\left(Vf^2 - v^2\right) \end{aligned}\right\} \quad \frac{dv}{dt} = \left(\frac{g}{Vf^2}\right)\cdot\left(Vf^2 - v^2\right)$$

$$\boxed{\begin{array}{l} g \text{ is a constant} \\ Vf \text{ is a constant} \end{array}}$$

$$dv = \left(\left(\frac{g}{Vf^2}\right)\cdot\left(Vf^2 - v^2\right)\right) dt$$

$$\frac{dv}{Vf^2 - v^2} = \frac{g}{Vf^2}\cdot dt \quad \leftarrow \quad \boxed{\begin{array}{l} a = Vf \\ b = 1 \end{array}}$$

$$\frac{1}{2Vf} \ln\left(\frac{Vf + v}{Vf - v}\right) = \frac{g}{Vf^2}\cdot t + C$$

$$t = \frac{Vf^2}{g}\cdot\frac{1}{2Vf} \ln\left(\frac{Vf + v}{Vf - v}\right) + C$$

$$t = \frac{Vf^2}{2g\,Vf} \ln\left(\frac{Vf + v}{Vf - v}\right) + C$$

$$t = \frac{Vf}{2g} \ln\left(\frac{Vf + v}{Vf - v}\right) + C$$

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$$\boxed{\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + C}$$